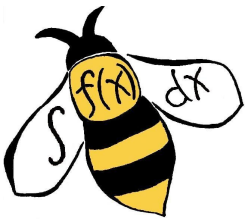
HIGH SCHOOL FINAL ROUND



The finals are conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

⇛

HIGH SCHOOL FINAL ROUND

Contestants must circle their final answer. Contestants do **not** need to include the constant of integration $+C$ in their answer.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,

GET SET,...

2:30

INTEGRAL #1

$$\int \left(x^2 + x\sqrt{2} + 1\right)\left(x^2 - x\sqrt{2} + 1\right)\,\mathrm{d}x$$

INTEGRAL #1

$$\int \left(x^2 + x\sqrt{2} + 1\right)\left(x^2 - x\sqrt{2} + 1\right) \, dx$$

$$= \int \left(x^4 + 1\right) \, dx$$

$$= \boxed{\frac{x^5}{5} + x}$$

INTEGRAL #2

READY,

GET SET,...

2:30

INTEGRAL #2

$$\int \sin x \cdot \sqrt[3]{4 + \cos x}\, dx$$

INTEGRAL #2

$$\int \sin x \cdot \sqrt[3]{4+\cos x}\,dx$$

$$= -\int \sqrt[3]{u}\,du \qquad u = 4+\cos x, \quad du = -\sin x\,dx$$

$$= -\frac{3u^{4/3}}{4}$$

$$= \boxed{-\frac{3(4+\cos x)^{4/3}}{4}}$$

INTEGRAL #3

READY,

GET SET,...

2:30

INTEGRAL #3

$$\int \frac{\sqrt[3]{x} \cdot \sqrt[4]{x}}{\sqrt[5]{x}} \, dx$$

INTEGRAL #3

$$\int \frac{\sqrt[3]{x} \cdot \sqrt[4]{x}}{\sqrt[5]{x}} \, dx$$

$$= \int \frac{x^{1/3} \cdot x^{1/4}}{x^{1/5}} \, dx$$

$$= \int \frac{x^{7/12}}{x^{1/5}} \, dx$$

$$= \int x^{23/60} \, dx$$

$$= \boxed{\frac{60x^{83/60}}{83}}$$

INTEGRAL #4

READY,

GET SET,...

2:30

INTEGRAL #4

$$\int \left(1+\sqrt[3]{x}\right)^3 \, \mathrm{d}x$$

INTEGRAL #4

$$\int \left(1+\sqrt[3]{x}\right)^3 \, dx$$

$$= \int \left(1+x^{1/3}\right)^3 \, dx$$

$$= \int \left(1+3x^{1/3}+3x^{2/3}+x\right) dx$$

$$= \boxed{x+\frac{9x^{4/3}}{4}+\frac{9x^{5/3}}{5}+\frac{x^2}{2}}$$

INTEGRAL #5

READY,

GET SET,...

2:30

INTEGRAL #5

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx$$

INTEGRAL #5

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx$$

$$= -\int \sec^2 u\,du \qquad u = \frac{1}{x}, \quad du = -\frac{1}{x^2}\,dx$$

$$= -\tan u$$

$$= \boxed{-\tan\left(\frac{1}{x}\right)}$$

INTEGRAL #6

READY,

GET SET,...

2:30

INTEGRAL #6

$$\int \frac{2e^{3x} + 4e^{5x} + 6e^{7x}}{e^{4x}} \, dx$$

INTEGRAL #6

$$\int \frac{2e^{3x} + 4e^{5x} + 6e^{7x}}{e^{4x}}\,dx$$

$$= \int \left(\frac{2e^{3x}}{e^{4x}} + \frac{4e^{5x}}{e^{4x}} + \frac{6e^{7x}}{e^{4x}} \right) dx$$

$$= \int \left(2e^{-x} + 4e^{x} + 6e^{3x}\right) dx$$

$$= \boxed{-2e^{-x} + 4e^{x} + 2e^{3x}}$$

INTEGRAL #7

READY,

GET SET,...

2:30

INTEGRAL #7

$$\int \frac{(x-6)^2}{x^3}\,dx$$

INTEGRAL #7

$$\int \frac{(x-6)^2}{x^3}\,dx$$

$$= \int \frac{x^2-12x+36}{x^3}\,dx$$

$$= \int \left(\frac{x^2}{x^3}-\frac{12x}{x^3}+\frac{36}{x^3}\right)dx$$

$$= \int \left(\frac{1}{x}-\frac{12}{x^2}+\frac{36}{x^3}\right)dx$$

$$= \boxed{\ln|x|+\frac{12}{x}-\frac{18}{x^2}}$$

INTEGRAL #8

READY,

GET SET,...

2:30

INTEGRAL #8

$$\int \sec 3x(\sec 3x + \tan 3x)\,\mathrm{d}x$$

INTEGRAL #8

$$\int \sec 3x(\sec 3x + \tan 3x)\,dx$$

$$= \int \left(\sec^2 3x + \sec 3x \tan 3x\right) dx$$

$$= \boxed{\tfrac{1}{3}\tan 3x + \tfrac{1}{3}\sec 3x}$$

INTEGRAL #9

READY,

GET SET,...

2:30

INTEGRAL #9

$$\int \frac{x}{e^{x^2}} \, dx$$

INTEGRAL #9

$$\int \frac{x}{e^{x^2}}\,dx$$

$$= \int x e^{-x^2}\,dx \qquad u = -x^2, \quad du = -2x\,dx$$

$$= -\frac{1}{2}\int e^{u}\,du$$

$$= -\frac{1}{2}e^{u}$$

$$= \boxed{-\frac{1}{2}e^{-x^2} = -\frac{1}{2e^{x^2}}}$$

INTEGRAL #10

READY,

GET SET,...

2:30

INTEGRAL #10

$$\int \frac{\sin \ln x}{x} \, dx$$

INTEGRAL #10

$$\int \frac{\sin \ln x}{x}\,dx$$

$$= \int \sin u\,du \qquad u = \ln x, \quad du = \frac{1}{x}\,dx$$

$$= -\cos u$$

$$= \boxed{-\cos \ln x}$$

INTEGRAL #11

READY,

GET SET,...

2:30

INTEGRAL #11

$$\int \frac{x^3}{\sqrt[3]{x^4+7}}\,dx$$

INTEGRAL #11

$$\int \frac{x^3}{\sqrt[3]{x^4+7}}\,dx$$

$$= \frac{1}{4}\int u^{-1/3}\,du \qquad u = x^4+7, \quad du = 4x^3\,dx$$

$$= \frac{3u^{2/3}}{8}$$

$$= \boxed{\frac{3(x^4+7)^{2/3}}{8}}$$

INTEGRAL #12

READY,

GET SET,...

2:30

INTEGRAL #12

$$\int \left(e^{2x} + \tan 2x\right)\left(e^{2x} + \sec^2 2x\right)\, dx$$

INTEGRAL #12

$$\int \left(e^{2x} + \tan 2x\right)\left(e^{2x} + \sec^2 2x\right) dx$$

$$= \frac{1}{2}\int u\, du \qquad u = e^{2x} + \tan 2x, \quad du = 2\left(e^{2x} + \sec^2 2x\right) dx$$

$$= \frac{u^2}{4}$$

$$= \boxed{\frac{\left(e^{2x} + \tan 2x\right)^2}{4}}$$

READY,

GET SET,...

2:30

INTEGRAL #13

$$\int (\mathrm{e}x^{\mathrm{e}} + x\mathrm{e}^{x})\,\mathrm{d}x$$

INTEGRAL #13

$$\int (\mathrm{e}x^{\mathrm{e}} + x\mathrm{e}^{x})\,\mathrm{d}x$$

$$= \frac{\mathrm{e}x^{\mathrm{e}+1}}{\mathrm{e}+1} + \int x\mathrm{e}^{x}\,\mathrm{d}x$$

$$= \frac{\mathrm{e}x^{\mathrm{e}+1}}{\mathrm{e}+1} + \left(x\mathrm{e}^{x} - \int \mathrm{e}^{x}\,\mathrm{d}x \right)$$

Integration by parts

$$= \boxed{\frac{\mathrm{e}x^{\mathrm{e}+1}}{\mathrm{e}+1} + x\mathrm{e}^{x} - \mathrm{e}^{x}}$$

INTEGRAL #14

READY,

GET SET,...

2:30

INTEGRAL #14

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^3}\,\mathrm{d}x$$

INTEGRAL #14

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^3}\,dx$$

$$= 2\int \frac{1}{u^3}\,du \qquad u = 1+\sqrt{x}, \quad du = \frac{1}{2\sqrt{x}}\,dx$$

$$= -\frac{1}{u^2}$$

$$= \boxed{-\frac{1}{\left(1+\sqrt{x}\right)^2}}$$

INTEGRAL #15

READY,

GET SET,...

2:30

INTEGRAL #15

$$\int \frac{e^{\sin x} + \sec^3 x}{\sec x} \, dx$$

INTEGRAL #15

$$\int \frac{e^{\sin x} + \sec^3 x}{\sec x}\,dx$$

$$= \int \left(\frac{e^{\sin x}}{\sec x} + \frac{\sec^3 x}{\sec x} \right) dx$$

$$= \int \left(e^{\sin x} \cos x + \sec^2 x \right) dx$$

$$= \boxed{e^{\sin x} + \tan x}$$

READY,

GET SET,...

2:30

INTEGRAL #16

$$\int \frac{\ln x}{x^2}\,dx$$

INTEGRAL #16

$$\int \frac{\ln x}{x^2}\,dx$$

Integrate by parts:

$$u = \ln x \qquad dv = \frac{1}{x^2}\,dx$$

$$du = \frac{1}{x}\,dx \qquad v = -\frac{1}{x}$$

$$= -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx$$

$$= \boxed{-\frac{\ln x}{x} - \frac{1}{x}}$$

INTEGRAL #17

READY,

GET SET,...

2:30

INTEGRAL #17

$$\int \frac{x - \sin 2x}{x^2 + \cos 2x} \, dx$$

INTEGRAL #17

$$\int \frac{x - \sin 2x}{x^2 + \cos 2x}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u}\,du \qquad u = x^2 + \cos 2x, \quad du = (2x - 2\sin 2x)\,dx = 2(x - \sin 2x)\,dx$$

$$= \frac{1}{2}\ln|u|$$

$$= \boxed{\frac{1}{2}\ln\left(x^2 + \cos 2x\right)}$$

READY,

GET SET,...

2:30

INTEGRAL #18

$$\int (x+1)\sec(x+1)\tan(x+1)\,dx$$

INTEGRAL #18

$$\int (x+1)\sec(x+1)\tan(x+1)\,dx$$

Integrate by parts: $u = x+1 \quad dv = \sec(x+1)\tan(x+1)\,dx$

$du = dx \quad v = \sec(x+1)$

$$= (x+1)\sec(x+1) - \int \sec(x+1)\,dx$$

$$= \boxed{(x+1)\sec(x+1) - \ln|\sec(x+1)+\tan(x+1)|}$$

INTEGRAL #19

READY,

GET SET,...

2:30

INTEGRAL #19

$$\int \frac{\tan^2 x}{\sec^2 x - \tan^2 x - \sin^2 x} \, dx$$

INTEGRAL #19

$$\int \frac{\tan^2 x}{\sec^2 x - \tan^2 x - \sin^2 x}\,dx$$

$$= \int \frac{\tan^2 x}{1 - \sin^2 x}\,dx \qquad \sec^2 x - \tan^2 x = 1$$

$$= \int \frac{\tan^2 x}{\cos^2 x}\,dx \qquad 1 - \sin^2 x = \cos^2 x$$

$$= \int \tan^2 x \sec^2 x\,dx = \int u^2\,du \qquad u = \tan x, \quad du = \sec^2 x\,dx$$

$$= \boxed{\frac{\tan^3 x}{3}}$$

INTEGRAL #20

READY,

GET SET,...

2:30

INTEGRAL #20

$$\int \frac{x}{\sqrt{2x-1}}\,\mathrm{d}x$$

INTEGRAL #20

$$\int \frac{x}{\sqrt{2x-1}}\,\mathrm{d}x$$

$$u=\sqrt{2x-1},\quad u^2=2x-1,\quad x=\frac{u^2+1}{2},\quad \mathrm{d}x=u\,\mathrm{d}u$$

$$=\int \frac{u^2+1}{2}\cdot\frac{1}{u}\cdot u\,\mathrm{d}u$$

$$=\frac{1}{2}\int \left(u^2+1\right)\mathrm{d}u$$

$$=\frac{1}{2}\left(\frac{u^3}{3}+u\right)=\boxed{\frac{1}{2}\left(\frac{\sqrt{(2x-1)^3}}{3}+\sqrt{2x-1}\right)}$$

INTEGRAL #21

READY,

GET SET,...

2:30

INTEGRAL #21

$$\int \frac{x^2+2x}{(x+1)^5}\,\mathrm{d}x$$

INTEGRAL #21

$$\int \frac{x^2+2x}{(x+1)^5}\,dx$$

$$= \int \left(\frac{x^2+2x+1}{(x+1)^5} - \frac{1}{(x+1)^5} \right) dx$$

$$= \int \left(\frac{(x+1)^2}{(x+1)^5} - \frac{1}{(x+1)^5} \right) dx$$

$$= \int \left(\frac{1}{(x+1)^3} - \frac{1}{(x+1)^5} \right) dx$$

$$= \boxed{-\frac{1}{2(x+1)^2} + \frac{1}{4(x+1)^4}}$$

INTEGRAL #22

READY,

GET SET,...

2:30

INTEGRAL #22

$$\int \frac{x^{\mathrm{e}-1} + \mathrm{e}^{x-1}}{x^{\mathrm{e}} + \mathrm{e}^{x}} \, \mathrm{d}x$$

INTEGRAL #22

$$\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x}\,dx$$

$$= \frac{1}{e}\int \frac{1}{u}\,du \qquad u = x^e + e^x, \quad du = (ex^{e-1} + e^x)\,dx$$
$$= e(x^{e-1} + e^{x-1})\,dx$$

$$= \frac{1}{e}\ln|u|$$

$$= \boxed{\frac{\ln|x^e+e^x|}{e}}$$

READY,

GET SET,...

2:30

INTEGRAL #23

$$\int \ln(x+\pi)\,\mathrm{d}x$$

INTEGRAL #23

$$\int \ln(x+\pi)\,dx$$

Integrate by parts:

$$u = \ln(x+\pi) \qquad dv = dx$$

$$du = \frac{1}{x+\pi}\,dx \qquad v = x+\pi$$

$$= (x+\pi)\ln(x+\pi) - \int 1\,dx$$

$$= \boxed{(x+\pi)\ln(x+\pi) - x}$$