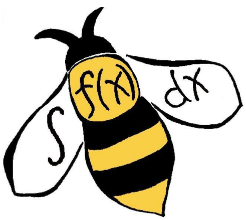
HIGH SCHOOL ROUND ONE

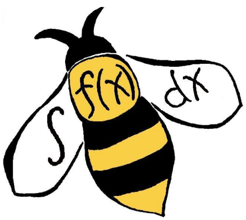


You will have **two minutes** to evaluate each of the fifteen definite integrals that will be displayed one at a time on this screen.

At the end of the two minutes, all hands must go up and judges will grade your answers immediately. If you wish to protest an answer, you must do so before the next problem is displayed by notifying a grader.

For each correct answer, you will receive one raffle ticket to be entered for prizes that will be drawn after dinner.

HIGH SCHOOL ROUND ONE



All answers must be simplified which means:

- Rational numbers must be in reduced form/simplified. BAD: $\frac{4}{2}$ vs GOOD: 2 or BAD: $\frac{1}{3}+\frac{1}{6}$ vs GOOD: $\frac{1}{2}$
- When irrational number(s) (e.g. e, π, $\ln 3$) are involved in fractions, you do NOT need to write as a single quantity and do not need to combine multiple logs.

  $\frac{1}{e}+2$ is just as good as $\frac{1+2e}{e}$, $2\ln 3$ is just as good as $\ln 9$

  However, NO negative powers should remain. So e^{-1} will be marked incorrect.
- All trig functions must be evaluated.

HIGH SCHOOL ROUND ONE

At most five participants will move to the finals – to be determined by the total number of correct answers and tiebreaking criteria if necessary.

Everyone moving to the finals will receive $25.

Questions?

INTEGRAL #1

READY,

GET SET,...

2:00

INTEGRAL #1

$$\int_0^1 \left(1600x^3 + 300x^2 + 1000x + 1017\right)\, dx$$

INTEGRAL #1

$$\int_0^1 \left(1600x^3 + 300x^2 + 1000x + 1017\right)\, dx$$

$$= \left[1600 \cdot \frac{x^4}{4} + 300 \cdot \frac{x^3}{3} + 1000 \cdot \frac{x^2}{2} + 1017x\right]_0^1$$

$$= \left[400x^4 + 100x^3 + 500x^2 + 1017x\right]_0^1$$

$$= \boxed{2017}$$

INTEGRAL #2

READY,

GET SET,...

2:00

INTEGRAL #2

$$\int_0^{\pi/4} (4\sin x + 6\cos x)\,dx$$

$$\int_0^{\pi/4} (4\sin x + 6\cos x)\,dx$$

$$= \Big[-4\cos x + 6\sin x\Big]_0^{\pi/4}\,dx$$

$$= \left(-4\cos\frac{\pi}{4} + 6\sin\frac{\pi}{4}\right) - (-4\cos 0 + 6\sin 0)$$

$$= \boxed{\sqrt{2} + 4}$$

INTEGRAL #3

READY,

GET SET,...

2:00

INTEGRAL #3

$$\int_1^2 \frac{3x^2 + 2}{x^2}\,dx$$

INTEGRAL #3

$$\int_1^2 \frac{3x^2+2}{x^2}\,dx$$

$$= \int_1^2 \left(\frac{3x^2}{x^2} + \frac{2}{x^2}\right)\,dx$$

$$= \int_1^2 \left(3 + 2x^{-2}\right) dx$$

$$= \left[3x - \frac{2}{x}\right]_1^2 = \boxed{4}$$

INTEGRAL #4

READY,

GET SET,...

2:00

INTEGRAL #4

$$\int_0^2 \sqrt{e^x}\,dx$$

INTEGRAL #4

$$\int_0^2 \sqrt{e^x}\, dx$$

$$= \int_0^2 (e^x)^{1/2}\, dx = \int_0^2 e^{x/2}\, dx$$

$$= 2\int_0^1 e^u\, du \qquad u = \frac{x}{2}, \quad du = \frac{1}{2}dx$$

$$= 2\left[e^u\right]_0^1$$

$$= \boxed{2(e-1) \quad \text{or} \quad 2e-2}$$

INTEGRAL #5

READY,

GET SET,...

2:00

INTEGRAL #5

$$\int_0^{\pi/4} \frac{\sec^2 x}{(1+\tan x)^3}\,dx$$

INTEGRAL #5

$$\int_0^{\pi/4} \frac{\sec^2 x}{(1+\tan x)^3}\,dx$$

$$= \int_1^2 \frac{1}{u^3}\,du \qquad u = 1+\tan x, \quad du = \sec^2 x\,dx$$

$$= \left[-\frac{1}{2u^2}\right]_1^2$$

$$= \boxed{\frac{3}{8}}$$

INTEGRAL #6

READY,

GET SET,...

2:00

INTEGRAL #6

$$\int_0^{13} \frac{1}{\sqrt[3]{2x+1}}\,dx$$

INTEGRAL #6

$$\int_0^{13} \frac{1}{\sqrt[3]{2x+1}}\,dx$$

$$= \frac{1}{2}\int_1^{27} u^{-1/3}\,du \qquad u = 2x+1, \quad du = 2\,dx$$

$$= \frac{3}{4}\left[u^{2/3}\right]_1^{27}$$

$$= \boxed{6}$$

INTEGRAL #7

READY,

GET SET,...

2:00

INTEGRAL #7

$$\int_0^1 (5x+3)(3x+5)\,dx$$

INTEGRAL #7

$$\int_0^1 (5x+3)(3x+5)\,dx$$

$$= \int_0^1 (15x^2 + 34x + 15)\,dx$$

$$= \left[15 \cdot \frac{x^3}{3} + 34 \cdot \frac{x^2}{2} + 15x\right]_0^1$$

$$= \left[5x^3 + 17x^2 + 15x\right]_0^1$$

$$= \boxed{37}$$

INTEGRAL #8

READY,

GET SET,...

2:00

INTEGRAL #8

$$\int_0^{\pi/3} (\sin x + \cos x)^2 \, dx$$

INTEGRAL #8

$$\int_0^{\pi/3} (\sin x + \cos x)^2 \, dx$$

$$= \int_0^{\pi/3} \left(\sin^2 x + 2\sin x \cos x + \cos^2 x\right) \, dx$$

$$= \int_0^{\pi/3} (1 + 2\sin x \cos x) \, dx$$

$$= \left[x + \sin^2 x\right]_0^{\pi/3} = \boxed{\frac{\pi}{3} + \frac{3}{4} \quad \text{or} \quad \frac{4\pi + 9}{12}}$$

INTEGRAL #9

READY,

GET SET,...

2:00

INTEGRAL #9

$$\int_0^1 \left(\sqrt[7]{x^4} - \sqrt[4]{x^7} \right) dx$$

INTEGRAL #9

$$\int_0^1 \left(\sqrt[7]{x^4} - \sqrt[4]{x^7}\right)\,dx$$

$$= \int_0^1 \left(x^{4/7} - x^{7/4}\right)\,dx$$

$$= \left[\frac{7x^{11/7}}{11} - \frac{4x^{11/4}}{11}\right]_0^1$$

$$= \boxed{\frac{3}{11}}$$

READY,

GET SET,...

2:00

INTEGRAL #10

$$\int_0^1 9xe^{3x}\,dx$$

INTEGRAL #10

$$\int_0^1 9xe^{3x}\,dx$$

integrate by parts: $u = 9x$ $dv = e^{3x}\,dx$

$du = 9\,dx$ $v = \frac{1}{3}e^{3x}$

$$= \left[3xe^{3x}\right]_0^1 - 3\int_0^1 e^{3x}\,dx$$

$$= \left[3xe^{3x} - e^{3x}\right]_0^1$$

$$= \boxed{2e^3 + 1}$$

INTEGRAL #11

READY,
GET SET,...

2:00

INTEGRAL #11

$$\int_1^2 (x+1)\sqrt{x-1}\,dx$$

INTEGRAL #11

$$\int_1^2 (x+1)\sqrt{x-1}\,dx$$

$$= \int_0^1 (u+2)\sqrt{u}\,du \quad u = x-1,\ \ u+2 = x+1,\ \ du = dx$$

$$= \int_0^1 \left(u^{3/2} + 2u^{1/2}\right)\,dx$$

$$= \left[\frac{2u^{5/2}}{5} + \frac{4u^{3/2}}{3}\right]_0^1 = \boxed{\frac{26}{15}}$$

READY,

GET SET,...

2:00

INTEGRAL #12

$$\int_0^2 x(x+3)^2\,dx$$

INTEGRAL #12

$$\int_0^2 x(x+3)^2\,dx$$

$$= \int_0^2 (x^3 + 6x^2 + 9x)\,dx$$

$$= \left[\frac{x^4}{4} + 2x^3 + \frac{9x^2}{2}\right]_0^2$$

$$= \boxed{38}$$

INTEGRAL #13

READY,

GET SET,...

2:00

INTEGRAL #13

$$\int_0^{\pi/2} \cos x \cdot \sin(\pi \sin x)\, dx$$

INTEGRAL #13

$$\int_0^{\pi/2} \cos x \cdot \sin(\pi \sin x)\, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \sin u\, du \qquad u = \pi \sin x, \quad du = \pi \cos x\, dx$$

$$= -\frac{1}{\pi}\Big[\cos u\Big]_0^{\pi}$$

$$= \boxed{\frac{2}{\pi}}$$

READY,

GET SET,...

2:00

INTEGRAL #14

$$\int_1^2 \frac{x+9}{\sqrt{x^2+18x-15}}\,dx$$

INTEGRAL #14

$$\int_1^2 \frac{x+9}{\sqrt{x^2+18x-15}}\,dx$$

$$= \frac{1}{2}\int_4^{25} u^{-1/2}\,du \qquad u = x^2+18x-15,\ \ du = 2(x+9)\,dx$$

$$= \left[u^{1/2}\right]_4^{25}$$

$$= \boxed{3}$$

INTEGRAL #15

READY,

GET SET,...

2:00

INTEGRAL #15

$$\int_0^{\pi/4} \frac{\sin^7 x}{\cos^9 x}\,dx$$

INTEGRAL #15

$$\int_0^{\pi/4} \frac{\sin^7 x}{\cos^9 x}\,dx$$

$$= \int_0^{\pi/4} \frac{\sin^7 x}{\cos^7 x} \cdot \frac{1}{\cos^2 x}\,dx = \int_0^{\pi/4} \tan^7 x \sec^2 x\,dx$$

$$= \int_0^1 u^7\,du \qquad u = \tan x, \quad du = \sec^2 x\,dx$$

$$= \left[\frac{u^8}{8}\right]_0^1 = \boxed{\frac{1}{8}}$$