THE HIGH SCHOOL FINALS

The finals are conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,
GET SET,...

2:30

INTEGRAL #1

$$\int \frac{\cos(1/x)}{x^2}\,dx$$

INTEGRAL #1

$$\int \frac{\cos(1/x)}{x^2}\,dx$$

$$= -\int \cos u\,du \qquad u = \frac{1}{x},\ du = -\frac{1}{x^2}\,dx$$

$$= -\sin u + C$$

$$= \boxed{-\sin\frac{1}{x} + C}$$

INTEGRAL #2

READY,
GET SET,...

2:30

INTEGRAL #2

$$\int \sqrt{x}(x^2 - 6)^2 \, dx$$

INTEGRAL #2

$$\int \sqrt{x}(x^2-6)^2\,dx$$

$$= \int \sqrt{x}(x^4-12x^2+36)\,dx$$

$$= \int \left(x^{9/2}-12x^{5/2}+36x^{1/2}\right)\,dx$$

$$= \boxed{\frac{2x^{11/2}}{11}-\frac{24x^{7/2}}{7}+24x^{3/2}+C}$$

INTEGRAL #3

READY,
GET SET,...

2:30

INTEGRAL #3

$$\int \frac{x+3}{(x^2+6x+7)^3}$$

INTEGRAL #3

$$\int \frac{x+3}{(x^2+6x+7)^3}$$

$$= \frac{1}{2}\int u^{-3}\,du \qquad u = x^2+6x+7,\ du = (2x+6)\,dx$$

$$= -\frac{1}{4u^2} + C$$

$$= \boxed{-\frac{1}{4(x^2+6x+7)^2} + C}$$

INTEGRAL #4

READY,
GET SET,...

2:30

INTEGRAL #4

$$\int \sin 5x \sqrt{\cos^3 5x}\, dx$$

INTEGRAL #4

$$\int \sin 5x \sqrt{\cos^3 5x}\, dx$$

$$= -\frac{1}{5}\int u^{3/2}\, du \qquad u = \cos 5x, \;\; du = -5 \sin 5x\, dx$$

$$= -\frac{1}{5} \cdot \frac{2u^{5/2}}{5} + C$$

$$= \boxed{-\frac{2}{25}\cos^{5/2} 5x + C}$$

INTEGRAL #5

READY,
GET SET,...

2:30

INTEGRAL #5

$$\int \frac{1}{x\sqrt{x}}\left(2+\frac{1}{\sqrt{x}}\right)^4 dx$$

INTEGRAL #5

$$\int \frac{1}{x\sqrt{x}}\left(2+\frac{1}{\sqrt{x}}\right)^4 dx$$

$$= -2\int u^4\, du \qquad u = 2+\frac{1}{\sqrt{x}},\ du = -\frac{1}{2x\sqrt{x}}\,dx$$

$$= -\frac{2u^5}{5} + C$$

$$= \boxed{-\frac{2}{5}\left(2+\frac{1}{\sqrt{x}}\right)^5 + C}$$

INTEGRAL #6

READY,
GET SET,...

2:30

INTEGRAL #6

$$\int \frac{\sin 2x}{\cos x}\,dx$$

INTEGRAL #6

$$\int \frac{\sin 2x}{\cos x}\,dx$$

$$= \int \frac{2\sin x \cos x}{\cos x}\,dx$$

$$= 2\int \sin x\,dx$$

$$= \boxed{-2\cos x + C}$$

INTEGRAL #7

READY,
GET SET,...

2:30

INTEGRAL #7

$$\int \frac{x}{(x+1)^4}\,dx$$

INTEGRAL #7

$$\int \frac{x}{(x+1)^4}\,dx$$

$$= \int \frac{u-1}{u^4}\,du \qquad u = x+1,\ x = u-1,\ dx = du$$

$$= \int (u^{-3} - u^{-4})\,du = -\frac{1}{2}u^{-2} + \frac{1}{3}u^{-3} + C$$

$$= \boxed{-\frac{1}{2(x+1)^2} + \frac{1}{3(x+1)^3} + C}$$

INTEGRAL #8

READY,
GET SET,...

2:30

INTEGRAL #8

$$\int \cot^3 x \sec^2 x \, dx$$

INTEGRAL #8

$$\int \cot^3 x \sec^2 x \, dx$$

$$= \int \frac{\cos x}{\sin^3 x} \, dx$$

$$= \int u^{-3} \, du \qquad u = \sin x, \;\; du = \cos x \, dx$$

$$= -\frac{1}{2u^2} + C = \boxed{-\frac{1}{2\sin^2 x} + C = -\frac{1}{2}\csc^2 x + C}$$

INTEGRAL #9

READY,
GET SET,...

2:30

INTEGRAL #9

$$\int \left(\frac{1}{x^2 + x} + \frac{1}{x + 1} \right) \, dx$$

INTEGRAL #9

$$\int \left(\frac{1}{x^2 + x} + \frac{1}{x + 1} \right) \, dx$$

$$= \int \frac{1}{x + 1} \left(\frac{1}{x} + 1 \right) \, dx$$

$$= \int \frac{1}{x} \, dx$$

$$= \boxed{\ln|x| + C}$$

INTEGRAL #10

READY,
GET SET,...

2:30

INTEGRAL #10

$$\int \frac{e^x + 2e^{2x} + 3e^{3x}}{4e^{4x}}\,dx$$

INTEGRAL #10

$$\int \frac{e^x + 2e^{2x} + 3e^{3x}}{4e^{4x}}\,dx$$

$$= \int \left(\frac{e^x}{4e^{4x}} + \frac{2e^{2x}}{4e^{4x}} + \frac{3e^{3x}}{4e^{4x}}\right)\,dx$$

$$= \int \left(\frac{1}{4}e^{-3x} + \frac{1}{2}e^{-2x} + \frac{3}{4}e^{-x}\right)\,dx$$

$$= \boxed{-\frac{1}{12}e^{-3x} - \frac{1}{4}e^{-2x} - \frac{3}{4}e^{-x} + C}$$

INTEGRAL #11

READY,
GET SET,...

2:30

INTEGRAL #11

$$\int \tan^2 \frac{x}{5} \sec^2 \frac{x}{5} \, dx$$

INTEGRAL #11

$$\int \tan^2\frac{x}{5}\sec^2\frac{x}{5}\,dx$$

$$= 5\int u^2\,du \qquad u = \tan\frac{x}{5},\ du = \frac{1}{5}\sec^2\frac{x}{5}\,dx$$

$$= \frac{5u^3}{3} + C$$

$$= \boxed{\frac{5}{3}\tan^3\frac{x}{5} + C}$$

INTEGRAL #12

READY,
GET SET,...

2:30

INTEGRAL #12

$$\int \frac{e^{2x}}{e^{4x} + 4e^{2x} + 4}\, dx$$

INTEGRAL #12

$$\int \frac{e^{2x}}{e^{4x}+4e^{2x}+4}\,dx$$

$$= \int \frac{e^{2x}}{(e^{2x}+2)^2}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u^2}\,du \qquad u = e^{2x}+2,\ \ du = 2e^{2x}\,dx$$

$$= -\frac{1}{2u} + C = \boxed{-\frac{1}{2(e^{2x}+2)} + C}$$

INTEGRAL #13

READY,
GET SET,...

2:30

INTEGRAL #13

$$\int \frac{x \ln(x^2+1)}{x^2+1}\,dx$$

INTEGRAL #13

$$\int \frac{x \ln(x^2+1)}{x^2+1}\,dx$$

$$= \frac{1}{2}\int u\,du \qquad u = \ln(x^2+1),\ du = \frac{2x}{x^2+1}\,dx$$

$$= \frac{u^2}{4} + C$$

$$= \boxed{\frac{1}{4}\left(\ln(x^2+1)\right)^2 + C}$$

INTEGRAL #14

READY,
GET SET,...

2:30

INTEGRAL #14

$$\int \frac{x}{\cos^2(x^2+3)}\,dx$$

INTEGRAL #14

$$\int \frac{x}{\cos^2(x^2+3)}\,dx$$

$$= \int x\sec^2(x^2+3)\,dx$$

$$= \frac{1}{2}\int \sec^2 u\,du \qquad u = x^2+3,\ \ du = 2x\,dx$$

$$= \frac{1}{2}\tan u + C = \boxed{\frac{1}{2}\tan(x^2+3) + C}$$

INTEGRAL #15

READY,
GET SET,...

2:30

INTEGRAL #15

$$\int \frac{2x-1}{\sqrt{x+3}}\,dx$$

INTEGRAL #15

$$\int \frac{2x-1}{\sqrt{x+3}}\,dx$$

$$= \int \frac{2(u-3)-1}{\sqrt{u}}\,du \qquad u = x+3,\ x = u-3,\ dx = du$$

$$= \int \left(2u^{1/2} - 7u^{-1/2}\right)\,du$$

$$= \boxed{\frac{4}{3}(x+3)^{3/2} - 14(x+3)^{1/2} + C}$$

INTEGRAL #16

READY,
GET SET,...

2:30

INTEGRAL #16

$$\int \frac{(1+e^x)^3}{e^x}\,dx$$

INTEGRAL #16

$$\int \frac{(1+e^x)^3}{e^x}\,dx$$

$$= \int \frac{1+3e^x+3e^{2x}+e^{3x}}{e^x}\,dx$$

$$= \int \left(e^{-x}+3+3e^x+e^{2x}\right)\,dx$$

$$= \boxed{-e^{-x}+3x+3e^x+\frac{e^{2x}}{2}+C}$$

INTEGRAL #17

READY,
GET SET,...

2:30

INTEGRAL #17

$$\int \frac{x+2}{e^{3x}}\,dx$$

INTEGRAL #17

$$\int \frac{x+2}{e^{3x}}\,dx$$

$$= -\frac{(x+2)e^{-3x}}{3} + \frac{1}{3}\int e^{-3x}\,dx \qquad \text{integrate by parts}$$

$$= -\frac{(x+2)e^{-3x}}{3} - \frac{e^{-3x}}{9} + C$$

$$= \boxed{-\frac{3x+7}{9e^{3x}} + C}$$