COLLEGE ROUND ONE

You will have **two minutes** to evaluate each of the fifteen definite integrals that will be displayed one at a time on this screen. **All answers must be simplified.** At the end of the two minutes, all hands must go up and judges will grade your answers immediately. For each correct answer, you will receive one raffle ticket to be entered for prizes that will be drawn after dinner.

At most five participants will move to the finals – to be determined by the total number of correct answers and tiebreaking criteria if necessary. **Everyone moving to the finals will receive $25.**

INTEGRAL #1

READY,
GET SET,...

2:00

INTEGRAL #1

$$\int_{-1}^{0} 3x^2(x^3+1)^{2016}\,dx$$

INTEGRAL #1

$$\int_{-1}^{0} 3x^2(x^3+1)^{2016}\,dx$$

$$= \int_0^1 u^{2016}\,du \qquad u = x^3+1,\ du = 3x^2\,dx$$

$$= \left[\frac{u^{2017}}{2017}\right]_0^1$$

$$= \boxed{\frac{1}{2017}}$$

INTEGRAL #2

READY,
GET SET,...

2:00

INTEGRAL #2

$$\int_0^{\pi/4} \frac{1 + 4\cos^2 x}{\cos^2 x}\, dx$$

INTEGRAL #2

$$\int_0^{\pi/4} \frac{1+4\cos^2 x}{\cos^2 x}\,dx$$

$$= \int_0^{\pi/4} \left(\sec^2 x + 4\right)\,dx$$

$$= \Big[\tan x + 4x\Big]_0^{\pi/4}$$

$$= \boxed{\pi + 1}$$

INTEGRAL #3

READY,
GET SET,...

2:00

INTEGRAL #3

$$\int_{10}^{11} x(x-10)^9 \, dx$$

INTEGRAL #3

$$\int_{10}^{11} x(x-10)^9\,dx$$

$$= \int_0^1 (u+10)u^9\,du \qquad u = x-10,\ du = dx$$

$$= \int_0^1 \left(u^{10}+10u^9\right)\,du$$

$$= \left[\frac{u^{11}}{11}+u^{10}\right]_0^1 = \boxed{\frac{12}{11}}$$

INTEGRAL #4

READY,
GET SET,...

2:00

INTEGRAL #4

$$\int_0^1 \left(\sqrt[7]{x\sqrt{x^5}} + \sqrt[5]{x\sqrt{x^3}} \right) \, dx$$

INTEGRAL #4

$$\int_0^1 \left(\sqrt[7]{x\sqrt{x^5}} + \sqrt[5]{x\sqrt{x^3}} \right) \, dx$$

$$= \int_0^1 \left(\left(x \cdot x^{5/2} \right)^{1/7} + \left(x \cdot x^{3/2} \right)^{1/5} \right) \, dx$$

$$= \int_0^1 \left(x^{1/2} + x^{1/2} \right) \, dx = 2 \int_0^1 x^{1/2} \, dx$$

$$= \left[\frac{4x^{3/2}}{3} \right]_0^1 = \boxed{\frac{4}{3}}$$

INTEGRAL #5

READY,
GET SET,...

2:00

INTEGRAL #5

$$\int_0^1 (x+1)e^{x+2}\,dx$$

INTEGRAL #5

$$\int_0^1 (x+1)e^{x+2}\,dx$$

integrate by parts: $u = x+1$ $\quad dv = e^{x+2}\,dx$

$du = dx$ $\quad v = e^{x+2}$

$$= \left[(x+1)e^{x+2}\right]_0^1 - \int_0^1 e^{x+2}\,dx$$

$$= \left[(x+1)e^{x+2} - e^{x+2}\right]_0^1 = \left[xe^{x+2}\right]_0^1 = \boxed{e^3}$$

INTEGRAL #6

READY,
GET SET,...

2:00

INTEGRAL #6

$$\int_0^1 \left((x^2+3)^2+(3x^2+1)^2\right)\,dx$$

INTEGRAL #6

$$\int_0^1 \left((x^2+3)^2+(3x^2+1)^2\right)\,dx$$

$$= \int_0^1 \left(10x^4+12x^2+10\right)\,dx$$

$$= \left[2x^5+4x^3+10x\right]_0^1$$

$$= \boxed{16}$$

INTEGRAL #7

READY,
GET SET,...

2:00

INTEGRAL #7

$$\int_0^4 \sqrt{x}\sqrt{1+x\sqrt{x}}\,dx$$

INTEGRAL #7

$$\int_0^4 \sqrt{x}\sqrt{1+x\sqrt{x}}\,dx$$

$$= \frac{2}{3}\int_1^9 \sqrt{u}\,du \qquad u = 1 + x\sqrt{x},\ du = \frac{3}{2}\sqrt{x}\,dx$$

$$= \left[\frac{4u^{3/2}}{9}\right]_1^9$$

$$= \boxed{\frac{104}{9}}$$

INTEGRAL #8

READY,
GET SET,...

2:00

INTEGRAL #8

$$\int_0^{\ln 3} \left(e^{x/2} + e^{-x/2}\right)^2 dx$$

INTEGRAL #8

$$\int_0^{\ln 3} \left(e^{x/2} + e^{-x/2}\right)^2 dx$$

$$= \int_0^{\ln 3} \left(e^{x} + 2 + e^{-x}\right)\, dx$$

$$= \left[e^{x} + 2x - e^{-x}\right]_0^{\ln 3}$$

$$= \boxed{\frac{8}{3} + 2\ln 3 = \frac{8}{3} + \ln 9}$$

INTEGRAL #9

READY,
GET SET,...

2:00

INTEGRAL #9

$$\int_{\pi/3}^{\pi} \frac{x^2 + \cos 3x}{x^3 + \sin 3x}\, dx$$

INTEGRAL #9

$$\int_{\pi/3}^{\pi} \frac{x^2 + \cos 3x}{x^3 + \sin 3x}\, dx$$

$$u = x^3 + \sin 3x, \quad du = (3x^2 + 3\cos 3x)\, dx$$

$$= \frac{1}{3}\int_{(\pi/3)^3}^{\pi^3} \frac{1}{u}\, du$$

$$= \frac{1}{3}\Big[\ln u\Big]_{(\pi/3)^3}^{\pi^3} = \boxed{\ln 3}$$

INTEGRAL #10

READY,
GET SET,...

2:00

INTEGRAL #10

$$\int_1^2 \frac{6}{x^2 - 3x}\,dx$$

INTEGRAL #10

$$\int_1^2 \frac{6}{x^2 - 3x}\,dx$$

$$= \int_1^2 \left(\frac{2}{x-3} - \frac{2}{x}\right) dx \qquad \text{partial fractions}$$

$$= \Big[2\ln|x-3| - 2\ln|x|\Big]_1^2$$

$$= \boxed{-4\ln 2 = -\ln 16 = \ln\frac{1}{16}}$$

INTEGRAL #11

READY,
GET SET,...

2:00

INTEGRAL #11

$$\int_{\sqrt{\pi}}^{\sqrt{4\pi/3}} x \sec(x^2 - \pi) \tan(x^2 - \pi)\, dx$$

INTEGRAL #11

$$\int_{\sqrt{\pi}}^{\sqrt{4\pi/3}} x \sec(x^2 - \pi) \tan(x^2 - \pi)\, dx$$

$$= \frac{1}{2} \int_0^{\pi/3} \sec u \tan u \, du \qquad u = x^2 - \pi, \;\; du = 2x\, dx$$

$$= \frac{1}{2} \Big[\sec u \Big]_0^{\pi/3}$$

$$= \boxed{\frac{1}{2}}$$

INTEGRAL #12

READY,
GET SET,...

2:00

INTEGRAL #12

$$\int_5^6 \frac{1}{x^2 - 10x + 26}\,dx$$

INTEGRAL #12

$$\int_5^6 \frac{1}{x^2 - 10x + 26}\,dx$$

$$= \int_5^6 \frac{1}{(x-5)^2 + 1}\,dx$$

$$= \Big[\arctan(x-5)\Big]_5^6$$

$$= \boxed{\frac{\pi}{4}}$$

INTEGRAL #13

READY,
GET SET,...

2:00

INTEGRAL #13

$$\int_0^{\pi/10} \sin^{10} 5x \cdot \cos 5x \, dx$$

INTEGRAL #13

$$\int_0^{\pi/10} \sin^{10} 5x \cdot \cos 5x \, dx$$

$$= \frac{1}{5} \int_0^1 u^{10} \, du \quad u = \sin 5x, \;\; du = 5 \cos 5x \, dx$$

$$= \left[\frac{u^{11}}{55} \right]_0^1$$

$$= \boxed{\frac{1}{55}}$$

INTEGRAL #14

READY,
GET SET,...

2:00

INTEGRAL #14

$$\int_0^{\pi/8} \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x \, dx$$

INTEGRAL #14

$$\int_0^{\pi/8} \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x \, dx$$

use $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$ repeatedly

$$= \frac{1}{2}\int_0^{\pi/8} \sin 2x \cdot \cos 2x \cdot \cos 4x \, dx = \frac{1}{4}\int_0^{\pi/8} \sin 4x \cos 4x \, dx$$

$$= \frac{1}{8}\int_0^{\pi/8} \sin 8x \, dx = \left[-\frac{\cos 8x}{64}\right]_0^{\pi/8} = \boxed{\frac{1}{32}}$$

INTEGRAL #15

READY,
GET SET,...

2:00

INTEGRAL #15

$$\int_2^3 \sqrt{1+\left(\frac{x^2}{2}-\frac{1}{2x^2}\right)^2}\,dx$$

INTEGRAL #15

$$\int_2^3 \sqrt{1+\left(\frac{x^2}{2}-\frac{1}{2x^2}\right)^2}\,dx$$

$$=\int_2^3 \sqrt{\frac{x^4}{4}+\frac{1}{2}+\frac{1}{4x^4}}\,dx$$

$$=\frac{1}{2}\int_2^3 \left(x^2+\frac{1}{x^2}\right)\,dx$$

$$=\frac{1}{2}\left[\frac{x^3}{3}-\frac{1}{x}\right]_2^3=\boxed{\frac{13}{4}}$$