THE HIGH SCHOOL FINALS

The finals are conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,

GET SET,...

2:30

INTEGRAL #1

$$\int (x^2+1)(x^3+3x)^4\,dx$$

2:30

INTEGRAL #1

$$\int (x^2+1)(x^3+3x)^4\,dx$$

$$\left[u = x^3+3x, \quad du = (3x^2+3)\,dx = 3(x^2+1)\,dx\right]$$

$$= \frac{1}{3}\int u^4\,du$$

$$= \frac{u^5}{15} + C = \boxed{\frac{(x^3+3x)^5}{15} + C}$$

INTEGRAL #2

READY,

GET SET,...

2:30

INTEGRAL #2

$$\int \frac{1}{\cos^2 x\sqrt{1+\tan x}}\,dx$$

2:30

INTEGRAL #2

$$\int \frac{1}{\cos^2 x\sqrt{1+\tan x}}\,dx$$

$$= \int \frac{\sec^2 x}{\sqrt{1+\tan x}}\,dx$$

$$= \int \frac{1}{\sqrt{u}}\,du \qquad \left[\, u = 1+\tan x, \quad du = \sec^2 x\,dx \,\right]$$

$$= 2\sqrt{u} + C = \boxed{2\sqrt{1+\tan x} + C}$$

INTEGRAL #3

READY,

GET SET,...

2:30

INTEGRAL #3

$$\int \left(\frac{2-x^2}{x^2}\right)^2 \mathrm{d}x$$

2:30

INTEGRAL #3

$$\int \left(\frac{2 - x^2}{x^2}\right)^2 dx$$

$$= \int \left(\frac{2}{x^2} - 1\right)^2 dx$$

$$= \int \left(\frac{4}{x^4} - \frac{4}{x^2} + 1\right) dx$$

$$= \boxed{-\frac{4}{3x^3} + \frac{4}{x} + x + C}$$

INTEGRAL #4

READY,

GET SET,...

2:30

INTEGRAL #4

$$\int x \cos^2(x^2) \sin(x^2)\, dx$$

2:30

INTEGRAL #4

$$\int x\cos^2\left(x^2\right)\sin\left(x^2\right)\,dx$$

$$= -\frac{1}{2}\int u^2\,du \qquad \left[\,u = \cos\left(x^2\right), \quad du = -2x\sin\left(x^2\right)\,dx\,\right]$$

$$= -\frac{u^3}{6} + C$$

$$= \boxed{-\frac{\cos^3\left(x^2\right)}{6} + C}$$

INTEGRAL #5

READY,

GET SET,...

2:30

INTEGRAL #5

$$\int (x+1)(e^x+1)\,dx$$

2:30

INTEGRAL #5

$$\int (x+1)(e^x+1)\,dx$$

$$\left[\text{integrate by parts: } \begin{array}{c} u = x+1 \\ du = dx \end{array}, \quad \begin{array}{c} dv = (e^x+1)\,dx \\ v = e^x + x \end{array}\right]$$

$$= (x+1)(e^x+x) - \int (e^x+x)\,dx$$

$$= (x+1)(e^x+x) - e^x - \frac{x^2}{2} + C = \boxed{xe^x + \frac{x^2}{2} + x + C}$$

INTEGRAL #6

READY,
GET SET,...

2:30

INTEGRAL #6

$$\int (x^2 + \sin 2x)(x + \cos 2x)\,dx$$

2:30

INTEGRAL #6

$$\int (x^2 + \sin 2x)(x + \cos 2x)\,dx$$

$$= \frac{1}{2}\int u\,du \quad \left[\, u = x^2 + \sin 2x, \quad du = (2x + 2\cos 2x)\,dx \,\right]$$

$$= \frac{u^2}{4} + C$$

$$= \boxed{\frac{(x^2 + \sin 2x)^2}{4} + C}$$

INTEGRAL #7

READY,

GET SET,...

2:30

INTEGRAL #7

$$\int x^2\sqrt{x+2}\,dx$$

2:30

INTEGRAL #7

$$\int x^2\sqrt{x+2}\,dx$$

$$= \int (u-2)^2\sqrt{u}\,du \quad [\, u = x+2, \quad du = dx \,]$$

$$= \int \left(u^{5/2} - 4u^{3/2} + 4u^{1/2}\right)\, du$$

$$= \boxed{\frac{2(x+2)^{7/2}}{7} - \frac{8(x+2)^{5/2}}{5} + \frac{8(x+2)^{3/2}}{3} + C}$$

INTEGRAL #8

READY,

GET SET,...

2:30

INTEGRAL #8

$$\int \frac{4x^2 + 4x + 5}{4x^2 + 4x + 1}\,dx$$

2:30

INTEGRAL #8

$$\int \frac{4x^2+4x+5}{4x^2+4x+1}\,dx$$

$$= \int \left(\frac{4x^2+4x+1}{4x^2+4x+1} + \frac{4}{4x^2+4x+1} \right) dx$$

$$= \int \left(1 + \frac{4}{(2x+1)^2} \right) dx$$

$$= \boxed{x - \frac{2}{2x+1} + C}$$

INTEGRAL #9

READY,

GET SET,...

2:30

INTEGRAL #9

$$\int \sqrt{1 + x^{-2/3}}\, dx$$

2:30

INTEGRAL #9

$$\int \sqrt{1 + x^{-2/3}}\, dx$$

$$= \int \sqrt{1 + \frac{1}{x^{2/3}}}\, dx = \int \sqrt{\frac{x^{2/3} + 1}{x^{2/3}}}\, dx = \int \frac{\sqrt{x^{2/3} + 1}}{x^{1/3}}\, dx$$

$$= \frac{3}{2} \int \sqrt{u}\, du \quad \left[u = x^{2/3} + 1, \quad du = \frac{2}{3} x^{-1/3}\, dx \right]$$

$$= \boxed{(x^{2/3} + 1)^{3/2} + C}$$