THE COLLEGE FINALS

The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,
GET SET,...

2:30

INTEGRAL #1

$$\int \frac{1+\sin x}{\cos x}\,dx$$

2:30

INTEGRAL #1

$$\int \frac{1+\sin x}{\cos x}\,dx$$

$$= \int \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)\,dx$$

$$= \int (\sec x + \tan x)\,dx$$

$$= \boxed{\ln|\sec x + \tan x| + \ln|\sec x| + C}$$

INTEGRAL #2

READY,
GET SET,...

2:30

INTEGRAL #2

$$\int \frac{x}{x^4+1}\,dx$$

2:30

INTEGRAL #2

$$\int \frac{x}{x^4+1}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u^2+1}\,du \qquad \left[\, u = x^2, \quad du = 2x\,dx \,\right]$$

$$= \frac{1}{2}\arctan u + C$$

$$= \boxed{\frac{\arctan\left(x^2\right)}{2} + C}$$

INTEGRAL #3

READY,
GET SET,...

2:30

INTEGRAL #3

$$\int \sin^4 x \cos^3 x \, dx$$

2:30

INTEGRAL #3

$$\int \sin^4 x \cos^3 x \, dx$$

$$= \int \sin^4 x \cos^2 x \cos x \, dx = \int \sin^4 x (1 - \sin^2 x) \cos x \, dx$$

$$= \int_0^1 (u^4 - u^6) \, du \quad [\, u = \sin x, \quad du = \cos x \, dx \,]$$

$$= \boxed{\frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C}$$

INTEGRAL #4

READY,
GET SET,...

2:30

INTEGRAL #4

$$\int \frac{\ln \ln x}{x \ln x}\, dx$$

2:30

INTEGRAL #4

$$\int \frac{\ln \ln x}{x \ln x}\, dx$$

$$= \int u\, du \qquad \left[u = \ln \ln x,\ du = \frac{1}{x \ln x}\, dx \right]$$

$$= \frac{u^2}{2} + C$$

$$= \boxed{\frac{(\ln \ln x)^2}{2} + C}$$

INTEGRAL #5

READY,
GET SET,...

2:30

INTEGRAL #5

$$\int \frac{x+1}{x^2+1}\,dx$$

2:30

INTEGRAL #5

$$\int \frac{x+1}{x^2+1}\,dx$$

$$= \int \left(\frac{x}{x^2+1} + \frac{1}{x^2+1} \right) dx$$

$$= \boxed{\frac{\ln(x^2+1)}{2} + \arctan x + C}$$

INTEGRAL #6

READY,
GET SET,...

2:30

INTEGRAL #6

$$\int \sec x \tan^5 x \, dx$$

2:30

INTEGRAL #6

$$\int \sec x \tan^5 x \, dx$$

$$= \int \tan^4 x \cdot \sec x \tan x \, dx = \int (\sec^2 x - 1)^2 \cdot \sec x \tan x \, dx$$

$$= \int (u^2 - 1)^2 \, du \quad [\, u = \sec x, \quad du = \sec x \tan x \, dx \,]$$

$$= \int (u^4 - 2u^2 + 1) \, du = \boxed{\frac{\sec^5 x}{5} - \frac{2\sec^3 x}{3} + \sec x + C}$$

INTEGRAL #7

READY,
GET SET,...

2:30

INTEGRAL #7

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$$

2:30

INTEGRAL #7

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$$

$$\left[x = \sin\theta, \quad dx = \cos\theta\,d\theta, \quad \sqrt{1-x^2} = \cos\theta\right]$$

$$= \int \frac{1}{\sin^2\theta\cos\theta}\cdot\cos\theta\,d\theta = \int \frac{1}{\sin^2\theta}\,d\theta$$

$$= \int \csc^2\theta\,d\theta = -\cot\theta + C = \boxed{-\frac{\sqrt{1-x^2}}{x} + C}$$

INTEGRAL #8

READY,
GET SET,...

2:30

INTEGRAL #8

$$\int \frac{x}{x^2 + x - 6}\, dx$$

2:30

INTEGRAL #8

$$\int \frac{x}{x^2+x-6}\,dx$$

$$= \int \left(\frac{3}{5}\cdot\frac{1}{x+3} + \frac{2}{5}\cdot\frac{1}{x-2}\right) dx \quad [\text{ partial fractions }]$$

$$= \boxed{\frac{3}{5}\ln|x+3| + \frac{2}{5}\ln|x-2| + C}$$

INTEGRAL #9

READY,
GET SET,...

2:30

INTEGRAL #9

$$\int \frac{\ln \tan x}{\sin x \cos x}\, dx$$

2:30

INTEGRAL #9

$$\int \frac{\ln \tan x}{\sin x \cos x}\, dx$$

$$\left[u = \ln \tan x, \quad du = \frac{\sec^2 x}{\tan x}\, dx = \frac{1}{\sin x \cos x}\, dx \right]$$

$$= \int u\, du$$

$$= \frac{u^2}{2} + C = \boxed{\frac{(\ln \tan x)^2}{2} + C}$$

INTEGRAL #10

READY,
GET SET,...

2:30

INTEGRAL #10

$$\int \frac{1 - e^{\tan x}}{1 - \sin^2 x} \, dx$$

2:30

INTEGRAL #10

$$\int \frac{1 - e^{\tan x}}{1 - \sin^2 x}\, dx$$

$$= \int \frac{1 - e^{\tan x}}{\cos^2 x}\, dx = \int (1 - e^{\tan x}) \sec^2 x\, dx$$

$$= \int (1 - e^{u})\, du \qquad [\, u = \tan x, \quad du = \sec^2 x\, dx \,]$$

$$= \boxed{\tan x - e^{\tan x} + C}$$

INTEGRAL #11

READY,
GET SET,...

2:30

INTEGRAL #11

$$\int \frac{1}{\sqrt{x}\sqrt{1-x}}\,dx$$

2:30

INTEGRAL #11

$$\int \frac{1}{\sqrt{x}\sqrt{1-x}}\,dx$$

$$= 2\int \frac{1}{\sqrt{1-u^2}}\,du \qquad \left[u = \sqrt{x}, \quad du = \frac{1}{2\sqrt{x}}\,dx \right]$$

$$= 2\arcsin u + C$$

$$= \boxed{2\arcsin\sqrt{x} + C}$$

INTEGRAL #12

READY,
GET SET,...

2:30

INTEGRAL #12

$$\int \frac{\ln x}{x + x\ln^2 x}\,dx$$

2:30

INTEGRAL #12

$$\int \frac{\ln x}{x + x\ln^2 x}\,dx$$

$$= \int \frac{\ln x}{x(1+\ln^2 x)}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u}\,du \quad \left[u = 1+\ln^2 x, \quad du = \frac{2\ln x}{x}\,dx\right]$$

$$= \frac{\ln u}{2} + C = \boxed{\frac{\ln\left(1+\ln^2 x\right)}{2} + C}$$

INTEGRAL #13

READY,
GET SET,...

2:30

INTEGRAL #13

$$\int \cos 2x \, (\sin x + \cos x)^2 \, dx$$

2:30

INTEGRAL #13

$$\int \cos 2x \,(\sin x + \cos x)^2 \, dx$$

$$= \int \cos 2x \left(\sin^2 x + 2 \sin x \cos x + \cos^2 x\right) \, dx$$

$$= \int \cos 2x \,(1 + 2 \sin x \cos x) \, dx = \int \cos 2x \,(1 + \sin 2x) \, dx$$

$$= \boxed{\frac{(1 + \sin x)^2}{4} \quad \text{or} \quad \frac{\sin 2x}{2} + \frac{\sin^2 2x}{4} \quad \text{or} \quad \frac{\sin 2x}{2} - \frac{\cos^2 2x}{4}}$$

INTEGRAL #14

READY,
GET SET,...

2:30

INTEGRAL #14

$$\int \frac{1}{(x-3)(x+1)+5}\,dx$$

2:30

INTEGRAL #14

$$\int \frac{1}{(x-3)(x+1)+5}\,dx$$

$$= \int \frac{1}{x^2-2x+2}\,dx$$

$$= \int \frac{1}{(x-1)^2+1}\,dx$$

$$= \boxed{\arctan(x-1)+C}$$

INTEGRAL #15

READY,
GET SET,...

2:30

INTEGRAL #15

$$\int \ln \sqrt{x^2 + 1}\, dx$$

2:30

INTEGRAL #15

$$\int \ln \sqrt{x^2+1}\, dx$$

$$= \frac{x \ln(x^2+1)}{2} - \int \frac{x^2}{x^2+1}\, dx \qquad [\text{ integrate by parts }]$$

$$= \frac{x \ln(x^2+1)}{2} - \int \left(1 - \frac{1}{x^2+1}\right) dx$$

$$= \boxed{\frac{x \ln(x^2+1)}{2} - x + \arctan x + C}$$