THE COLLEGE FINALS

The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,
GET SET,...

2:30

INTEGRAL #1

$$\int \frac{\ln x}{x^2}\, dx$$

2:30

INTEGRAL #1

$$\int \frac{\ln x}{x^2}\,dx$$

$$\left[\text{integrate by parts: } \begin{array}{rcl} u &=& \ln x \\ du &=& \frac{1}{x}\,dx \end{array}, \quad \begin{array}{rcl} dv &=& \frac{1}{x^2}\,dx \\ v &=& -\frac{1}{x} \end{array}\right]$$

$$= -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x} + C$$

$$= \boxed{-\frac{1+\ln x}{x} + C}$$

INTEGRAL #2

READY,
GET SET,...

2:30

INTEGRAL #2

$$\int \left(e^{x} + e^{-x}\right)^{2} \, dx$$

2:30

INTEGRAL #2

$$\int \left(e^x + e^{-x}\right)^2 dx$$

$$= \int \left(e^{2x} + 2 + e^{-2x}\right)$$

$$= \frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2} + C$$

$$= \boxed{\frac{4x + e^{2x} - e^{-2x}}{2} + C}$$

INTEGRAL #3

READY,
GET SET,...

2:30

INTEGRAL #3

$$\int e^{2x}\sqrt{1+e^{2x}}\,dx$$

2:30

INTEGRAL #3

$$\int e^{2x}\sqrt{1+e^{2x}}\,dx$$

$$= \frac{1}{2}\int \sqrt{u}\,du \quad \left[u = 1+e^{2x}, \quad du = 2e^{2x}\,dx\right]$$

$$= \frac{u^{3/2}}{3} + C$$

$$= \boxed{\frac{\left(1+e^{2x}\right)^{3/2}}{3} + C}$$

INTEGRAL #4

READY,
GET SET,...

2:30

INTEGRAL #4

$$\int \frac{\ln \sqrt{x}}{x}\, dx$$

2:30

INTEGRAL #4

$$\int \frac{\ln \sqrt{x}}{x}\, dx$$

$$= \int \frac{\ln x^{1/2}}{x}\, dx = \frac{1}{2}\int \frac{\ln x}{x}\, dx$$

$$= \frac{1}{2}\int u\, du \qquad \left[u = \ln x, \quad du = \frac{1}{x}\, dx\right]$$

$$= \frac{u^2}{4} = \boxed{\frac{(\ln x)^2}{4} + C}$$

INTEGRAL #5

READY,
GET SET,...

2:30

INTEGRAL #5

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\, dx$$

2:30

INTEGRAL #5

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u}\,du \quad \left[u = e^{2x} + e^{-2x}, \quad du = \left(2e^{2x} - 2e^{-2x}\right)dx\right]$$

$$= \frac{1}{2}\ln u + C$$

$$= \boxed{\frac{\ln\left(e^{2x} + e^{-2x}\right)}{2} + C}$$

INTEGRAL #6

READY,
GET SET,...

2:30

INTEGRAL #6

$$\int x \sec^2 x \, dx$$

2:30

INTEGRAL #6

$$\int x \sec^2 x \, dx$$

$$\left[\text{integration by parts: } \begin{array}{rcl} u &=& x \\ du &=& dx \end{array}, \quad \begin{array}{rcl} dv &=& \sec^2 x \, dx \\ v &=& \tan x \end{array} \right]$$

$$= x \tan x - \int \tan x \, dx$$

$$= \boxed{x \tan x + \ln|\cos x| + C}$$

INTEGRAL #7

READY,
GET SET,...

2:30

INTEGRAL #7

$$\int x\left(1+\frac{1}{x}\right)^3\,dx$$

2:30

INTEGRAL #7

$$\int x\left(1+\frac{1}{x}\right)^3\,dx$$

$$=\int x\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)\,dx$$

$$=\int\left(x+3+\frac{3}{x}+\frac{1}{x^2}\right)\,dx$$

$$=\boxed{\frac{x^2}{2}+3x+3\ln x-\frac{1}{x}+C}$$

INTEGRAL #8

READY,
GET SET,...

2:30

INTEGRAL #8

$$\int \cos^5 x \, dx$$

2:30

INTEGRAL #8

$$\int \cos^5 x \, dx$$

$$= \int (\cos^2 x)^2 \cos x \, dx = \int (1 - \sin^2 x)^2 \cos x \, dx$$

$$= \int (1 - u^2)^2 \, du \quad [u = \sin x, \quad du = \cos x \, dx]$$

$$= \int (u^4 - 2u^2 + 1) \, du = \boxed{\frac{\sin^5 x}{5} - \frac{2\sin^3 x}{3} + \sin x + C}$$

INTEGRAL #9

READY,
GET SET,...

2:30

INTEGRAL #9

$$\int \frac{x^2+1}{(x+1)^2}\,dx$$

2:30

INTEGRAL #9

$$\int \frac{x^2+1}{(x+1)^2}\,dx$$

$$= \int \frac{(u-1)^2+1}{u^2}\,du \quad [u = x+1, \quad du = dx, \quad x = u-1]$$

$$= \int \frac{u^2-2u+2}{u^2}\,du = \int \left(1 - \frac{2}{u} + \frac{2}{u^2}\right)\,du$$

$$= u - 2\ln|u| - \frac{2}{u} + C = \boxed{x+1-2\ln|x+1| - \frac{2}{x+1} + C}$$

INTEGRAL #10

READY,
GET SET,...

2:30

INTEGRAL #10

$$\int \frac{x^2}{x-1}\,dx$$

2:30

INTEGRAL #10

$$\int \frac{x^2}{x-1}\,dx$$

$$= \int \left(x + 1 + \frac{1}{x-1}\right)\,dx$$

$$= \boxed{\frac{x^2}{2} + x + \ln|x-1| + C}$$

INTEGRAL #11

READY,
GET SET,...

2:30

INTEGRAL #11

$$\int \frac{1}{\sqrt{x^2-1}}\,dx$$

2:30

INTEGRAL #11

$$\int \frac{1}{\sqrt{x^2-1}}\,dx$$

$$\left[\, x = \sec\theta, \quad dx = \sec\theta\tan\theta\,d\theta, \quad \sqrt{x^2-1} = \tan\theta \,\right]$$

$$= \int \frac{\sec\theta\tan\theta}{\tan\theta}\,d\theta = \int \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C = \boxed{\ln|x + \sqrt{x^2-1}| + C}$$

INTEGRAL #12

READY,
GET SET,...

2:30

INTEGRAL #12

$$\int \sin^2 x \cos^2 x \, dx$$

2:30

INTEGRAL #12

$$\int \sin^2 x \cos^2 x \, dx$$

$$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx$$

$$= \frac{1}{4}\int (1-\cos^2 2x)\, dx = \frac{1}{4}\int \sin^2 2x \, dx = \frac{1}{4}\int \left(\frac{1-\cos 4x}{2}\right)$$

$$= \frac{1}{8}\int (1-\cos 4x)\, dx = \boxed{\frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) + C}$$