THE COLLEGE FINALS

The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,

GET SET,...

2:30

INTEGRAL #1

$$\int \frac{x}{e^{x^2}}\,dx$$

2:30

INTEGRAL #1

$$\int \frac{x}{e^{x^2}}\,dx$$

$$= \int x e^{-x^2}\,dx$$

$$= -\frac{1}{2}\int e^{u}\,du \qquad \left[u = -x^2, \quad du = -2x\,dx\right]$$

$$= -\frac{e^{u}}{2} + C = \boxed{-\frac{e^{-x^2}}{2} + C \quad \text{or} \quad -\frac{1}{2e^{x^2}} + C}$$

INTEGRAL #2

READY,

GET SET,...

2:30

INTEGRAL #2

$$\int \frac{x}{x^4 + 6x^2 + 9}\,dx$$

2:30

INTEGRAL #2

$$\int \frac{x}{x^4 + 6x^2 + 9}\,dx$$

$$= \int \frac{x}{(x^2+3)^2}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u^2}\,du \qquad [u = x^2 + 3, \quad du = 2x\,dx]$$

$$= \frac{1}{2} \cdot \frac{-1}{u} + C = \boxed{-\frac{1}{2(x^2+3)} + C}$$

INTEGRAL #3

READY,

GET SET,...

2:30

INTEGRAL #3

$$\int \frac{x+4}{x^2+4}\,dx$$

2:30

INTEGRAL #3

$$\int \frac{x+4}{x^2+4}\,dx$$

$$= \int \left(\frac{x}{x^2+4} + \frac{4}{x^2+4} \right)\,dx$$

$$= \boxed{\frac{1}{2}\ln(x^2+4) + 2\arctan\frac{x}{2} + C}$$

INTEGRAL #4

READY,

GET SET,...

2:30

INTEGRAL #4

$$\int \frac{\ln x}{\sqrt{x}}\,dx$$

2:30

INTEGRAL #4

$$\int \frac{\ln x}{\sqrt{x}}\,dx$$

$$= 2\sqrt{x}\ln x - \int 2\sqrt{x}\cdot\frac{1}{x}\,dx \qquad \left[\text{by parts: } u = \ln x, \quad dv = \frac{dx}{\sqrt{x}}\right]$$

$$= 2\sqrt{x}\ln x - 2\int \frac{1}{\sqrt{x}}\,dx$$

$$= \boxed{2\sqrt{x}\ln x - 4\sqrt{x} + C}$$

INTEGRAL #5

READY,

GET SET,...

2:30

INTEGRAL #5

$$\int \frac{\sqrt{x}}{\sqrt{x\sqrt{x}+\sqrt{7}}}\,dx$$

2:30

INTEGRAL #5

$$\int \frac{\sqrt{x}}{\sqrt{x\sqrt{x}+\sqrt{7}}}\,dx$$

$$= \frac{2}{3}\int \frac{1}{\sqrt{u}}\,du \qquad \left[u = x\sqrt{x}+7, \quad du = \frac{3}{2}\sqrt{x}\right]$$

$$= \frac{2}{3}\cdot 2\sqrt{u} = \boxed{\frac{4}{3}\sqrt{x\sqrt{x}+\sqrt{7}}}$$

INTEGRAL #6

READY,

GET SET,...

2:30

INTEGRAL #6

$$\int \frac{x^2}{x^2+1}\,dx$$

2:30

INTEGRAL #6

$$\int \frac{x^2}{x^2+1}\,dx$$

$$= \int \frac{x^2+1-1}{x^2+1}\,dx = \int \left(\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\right)\,dx$$

$$= \int \left(1 - \frac{1}{x^2+1}\right)\,dx$$

$$= \boxed{x - \arctan x + C}$$

INTEGRAL #7

READY,

GET SET,...

2:30

INTEGRAL #7

$$\int x \sec x \tan x \, dx$$

2:30

INTEGRAL #7

$$\int x \sec x \tan x \, dx$$

$$\left[\text{integration by parts: } \begin{aligned} u &= x \\ du &= dx \end{aligned}, \quad \begin{aligned} dv &= \sec x \tan x \, dx \\ v &= \sec x \end{aligned} \right]$$

$$= x \sec x - \int \sec x \, dx$$

$$= \boxed{x \sec x - \ln|\sec x + \tan x| + C}$$

INTEGRAL #8

READY,

GET SET,...

2:30

INTEGRAL #8

$$\int x\sqrt{x+5}\,dx$$

2:30

INTEGRAL #8

$$\int x\sqrt{x+5}\,dx$$

$$= \int (u-5)\sqrt{u}\,du \quad \left[u = x+5, \quad du = dx\right]$$

$$= \int \left(u^{3/2} - 5u^{1/2}\right)\,du = \frac{2u^{5/2}}{5} - 5\cdot\frac{2u^{3/2}}{3} + C$$

$$= \boxed{\frac{2(x+5)^{5/2}}{5} - \frac{10(x+5)^{3/2}}{3} + C}$$

INTEGRAL #9

READY,

GET SET,...

2:30

INTEGRAL #9

$$\int (\sec x + \tan x)^2 \, dx$$

2:30

INTEGRAL #9

$$\int (\sec x + \tan x)^2 \, dx$$

$$= \int \left(\sec^2 x + 2\sec x \tan x + \tan^2 x\right) \, dx$$

$$= \int \left(\sec^2 x + 2\sec x \tan x + \sec^2 x - 1\right) \, dx$$

$$= \int \left(2\sec^2 x + 2\sec x \tan x - 1\right) \, dx$$

$$= \boxed{2\tan x + 2\sec x - x + C}$$

INTEGRAL #10

READY,

GET SET,...

2:30

INTEGRAL #10

$$\int \frac{dx}{x^2 + 2x + 10}$$

2:30

INTEGRAL #10

$$\int \frac{dx}{x^2 + 2x + 10}$$

$$= \int \frac{dx}{(x+1)^2 + 9}$$

$$= \int \frac{du}{u^2 + 3^2} \qquad [u = x + 1, \quad du = dx]$$

$$= \frac{1}{3} \arctan \frac{u}{3} + C = \boxed{\frac{1}{3} \arctan \frac{x+1}{3} + C}$$

INTEGRAL #11

READY,

GET SET,...

2:30

INTEGRAL #11

$$\int \frac{dx}{\sqrt{x+1}+\sqrt{x-1}}$$

2:30

INTEGRAL #11

$$\int \frac{dx}{\sqrt{x+1}+\sqrt{x-1}}$$

$$= \int \frac{1}{\sqrt{x+1}+\sqrt{x-1}} \cdot \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\,dx$$

$$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{2}\,dx$$

$$= \boxed{\frac{(x+1)^{3/2}-(x-1)^{3/2}}{3}+C}$$

INTEGRAL #12

READY,

GET SET,...

2:30

INTEGRAL #12

$$\int \arctan \frac{1}{x}\, dx$$

2:30

INTEGRAL #12

$$\int \arctan\frac{1}{x}\,dx$$

$$\left[\text{integrate by parts: } \begin{array}{rcl} u &=& \arctan\frac{1}{x} \\ du &=& -\frac{1}{x^2+1}\,dx \end{array}, \quad \begin{array}{rcl} dv &=& dx \\ v &=& x \end{array}\right]$$

$$= x\arctan\frac{1}{x} + \int \frac{x}{x^2+1}\,dx$$

$$= \boxed{x\arctan\frac{1}{x} + \frac{1}{2}\ln(x^2+1) + C}$$